PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Cauchy theorem.

17. State and prove the Eisenstein criterion for polynomial rings.

18. State and prove Gram-Schmidt orthogonalization process.

19. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover, $[L : F] = [L : K] [K: F]$.

20. If K is a finite extension of F, then prove that $G (K, F)$ is a finite group and its order, $o(G (K, F))$ satisfies $o(G(K,F)) \leq [K :F]$.

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S.No. 167 **12 PMA 01**

(For the candidates admitted from 2012 – 2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

First Semester

Mathematics

ALGEBRA

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. State first Sylow's theorem.

2. Define equivalence relation in group.

3. Define unique factorization domain.

4. Define irreducible polynomial.

5. Define dual space.

6. Define inner product space.

7. When an element is said to be algebraic over F.

8. Define simple extension of a field F.

9. Define Galois group.

10. Define normal extension of field F.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL the questions.

11. (a) If $o(G) = p^2$ where p is a prime number then prove that the group G is abelian.

Or

(b) Show that conjugacy is an equivalence relation in group G.

12. (a) If $f(x)$ and $g(x)$ are primitive polynomials, then prove that $f(x)\, g(x)$ is a primitive polynomial.

Or

(b) State and prove Gauss lemma.

13. (a) State and prove Schwarz inequality.

Or

(b) If V is a finite-dimensional inner product space and if W is a subspace of V, then prove that $V = W + W^{\perp}$.

14. (a) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

Or

(b) State and prove the Remainder theorem.

15. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) Let $G = S_n$, were $n \geq 5$; the prove that $G^{(K)}$ for $K = 1,2,....,$ contains every 3-cycle of S_n.